

10-29-09

Jean Francois Dat Part 1: Cohomological Methods in Langlands Program

Let \mathcal{H} = upper half plane

We want to introduce a p-adic analogue of \mathcal{H} .

Well, recall $\mathcal{H} = SL(2, \mathbb{R}) / SO(2, \mathbb{R})$

= homogeneous space. But

You don't see \mathcal{H} as a complex mfd this way.

Also, $\mathcal{H} = \{z : \text{Im}(z) > 0\}$, so you can

see it as a complex mfd.

Also, $\mathcal{H} =$ open unit disk.

$SL_2(\mathbb{R}) / SO(2, \mathbb{R})$ is a Riemannian symmetric space,

and is geodesically complete.

Maire guess :

From $SL(2, \mathbb{R}) / SO(2, \mathbb{R})$, try to define

$$GL(2, \mathbb{Q}_p) / GL(2, \mathbb{Z}/p) \cdot \mathbb{Q}_p^\times$$

Note: 1) $GL(2, \mathbb{Z}/p)$ cpt \Rightarrow is discrete
This problem

$$2) \quad \mathbb{H}^{\pm} = GL(2, \mathbb{R}) / O(2, \mathbb{R}) \cdot \mathbb{R}^\times$$

We will now define the Bruhat-Tits building in terms of lattices, for the rest of this ~~talk~~ talk.

Let $V =$ vector space / \mathbb{Q}_p of dim $= n$.

~~(*)~~ A lattice is a fin. gen. \mathbb{Z}_p -submodule

\mathbb{Z}_p^n
 \mathbb{Z}_p
 $W \subset V$ which generates V over \mathbb{Q}_p .

Let $G = GL_n(\mathbb{Q}_p) = GL(V)$
 \uparrow
 \mathbb{Q}_p

Defn: $B_0(V) := \{ \text{homothety classes of lattices} \}$
 $=_q G$ -transitive set

Then $B_0(V) \cong \frac{GL(V)}{GL(W) \cdot Z}$

where $Z = \text{center of } GL_n(\mathbb{Q}_p)$.

For $m \geq 0$, define $B_m^{abs}(V) := \left\{ \begin{array}{l} \mathcal{Z} \rightarrow \{ \text{lattices} \} \\ i \mapsto w(i) \end{array} \right.$

non-increasing such that $w(i+m+1) = pw(i)$

where ~~$f(\cdot)$~~

$$f(\cdot) \sim f(\cdot + m + 1)$$

$$\int_0 B_0^{abs}(V) = B_0(V)$$

Order preserving map

$$\{0, \dots, m-1\} \rightarrow \{0, \dots, m'-1\}$$

indices

$$B_{m'}^{abs}(V) \rightarrow B_m^{abs}(V)$$

$$W' \xrightarrow{\omega} W : i \mapsto p^n w' (2r)$$

$$\cong \mathbb{K}(m, r)$$

$$\text{where } i = k(m'+1) + r$$

Then $B_{\bullet}^{abs}(V) = \text{simplicial set } \bullet$

\uparrow dot, i.e. the collection of all $B_i^{abs}(V)$

where simplicial set means a functor
from the category of \mathbb{N} sets to
the category of \mathbb{N} sets

(this is the topologist's defn of
simplicial set)

From this geometric realization, we get

a "simplicial complex" called $B_{\bullet}(V)$

\uparrow dot

~~Let~~

Let $B_m(V) = \{ m\text{-simplices of } B_0(V) \}$

$= \{ \text{all increasing } w \text{ in } B_0^{qs}(V) \}$

~~f~~ \sim

where

$f(\cdot) \sim f(\cdot+1)$



$w(0) + \text{flag in } w(0) / \underbrace{pw(0)}$

$n\text{-dim'l vector space over } \mathbb{F}_p$

Then $B_0(V)$ is $(n-1)\text{-dim'l and homogeneous.}$

Ex: If $n=2$, get graph

w a lattice, then

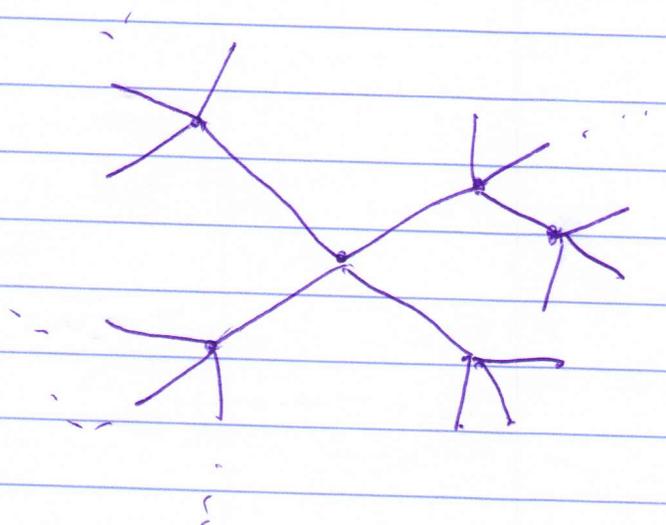
$$\{\text{neighbours of } w\} \longleftrightarrow \left\{ \begin{array}{l} \text{lines in} \\ w \otimes \mathbb{F}_p \end{array} \right\}$$

\cong

$$\mathbb{P}^1(\mathbb{F}_p)$$

\uparrow cardinality $p+1$

So if $p=3$, get a tree



There is an alternative definition in terms of norms (probably just like in JK Yu's notes on buildings) as follows:

Let $\mathcal{N}(V) := \{ \text{homothety classes of norms} \}$

Start with a norm $\|\cdot\|$. ~~Get~~
 Associated to $\|\cdot\|$, we get a lattice function

$$\omega_{\|\cdot\|} : \mathbb{R} \longrightarrow \{ \text{lattices} \}$$

$$x \longmapsto \{ v, \|v\| \leq p^{-x} \}$$

This is a non-increasing function which is lower semi-continuous, and $\omega_{\|\cdot\|}(x+1) = p \omega_{\|\cdot\|}(x)$

This all implies ~~that~~ that we can choose a map

$$j : \mathbb{Z} \xrightarrow{\sim} \{ \text{jumps of } \omega_{\|\cdot\|} \}$$

that is order preserving bijection.

$$\omega_{\|\cdot\|} \circ j \in B_m^{\text{abs}}(V) \quad m = \# \{ \text{jumps} \in [0, 1] \}$$

$$\text{Let } d_i = j(i) - j(i-1)$$

$$\text{Then } \sum_{i=0}^{m-1} d_i = 1.$$

$$\text{Prop: } N(V) \longrightarrow B(V)$$

$\|\cdot\| \longleftrightarrow$ the unique point with barycentric coordinates (d_i) in simplex $\omega \circ j$.

is a homeomorphism

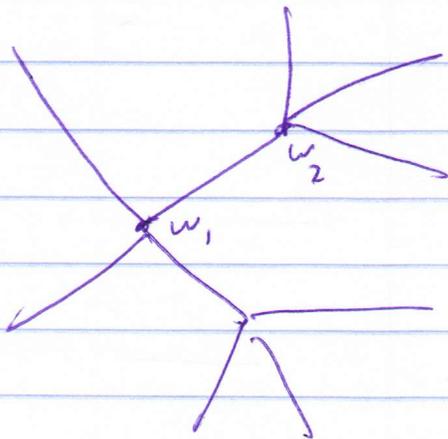
Dynamics of $GL_2(\mathbb{Q}_p)$ on the tree ^{action}

Stabilizers: $\text{Stab}_G(x)$ for x a vertex,

is $GL(w)\mathbb{Q}_p^*$, $\varphi \in w \subset V$.

If $[w_1, w_2]$ is an edge,

So



and we write

$$w_1 = \mathbb{Z}_p e_1 \oplus \mathbb{Z}_p e_2,$$

$$w_2 = p\mathbb{Z}_p e_1 \oplus \mathbb{Z}_p e_2,$$

then if x is in the interior

of the edge, then

$$\text{Stab}_G(x) = \begin{cases} I \oplus_{\mathbb{Z}_p}^x & \text{where } I = I_{w_1, w_2} \\ & \text{subgroup of } GL_2(\mathbb{Z}_p). \\ & \text{if } x \text{ is not the center of the edge} \\ \text{Normalizer}(I) = I \oplus_{\mathbb{Z}_p}^x \perp I \begin{pmatrix} 0 & p \\ 0 & 1 \end{pmatrix} \oplus_{\mathbb{Z}_p}^x & \text{if } x \text{ is the center of the edge } [w_1, w_2]. \end{cases}$$

Apartments / Action of maximal split tori.

Choose $\{e_1, e_2\}$
a basis of V , $GL(V) = GL_2(\mathbb{Q}_p)$,

$T =$ split max'l torus.

Let $A_T := \{ \|\cdot\| \in \mathcal{N}(V) \text{ s.t.}$

$$\|v\| := \max\{\|v_1\|_{e_1}, \|v_2\|_{e_2}\}$$

where $v = v_1 e_1 + v_2 e_2$

$\|v_1\|_{e_1}$



$=$ convex hull of

$$\left\{ w \in B_0(V) \text{ s.t. } \exists i \in \mathbb{Z} \right. \\ \left. \text{with } w = p^i z_p e_1 \oplus p^j z_p e_2 \right\}$$

If $g \in G$, then the action of

g takes A_T to $A_{gTg^{-1}}$

So $N_G(T)$ acts on A_T .

This action is trivial on $T^0 =$ maximal compact subgroup of T .

The action factors through

$$\frac{N_G(T)}{T^0 Z} \quad \text{where } Z = \text{center.}$$

||
Affine Weyl group.

Then, get $A_T \cong \frac{T}{T^0 Z} \otimes \mathbb{R} \cong X_*(T/Z) \otimes \mathbb{R}$

Action of unipotent and Borel subgroups:

$$\text{Let } U = \left\{ \begin{pmatrix} 1 & * \\ & 1 \end{pmatrix} \right\}, B = \left\{ \begin{pmatrix} * & * \\ & * \end{pmatrix} \right\}$$

in GL_2 .

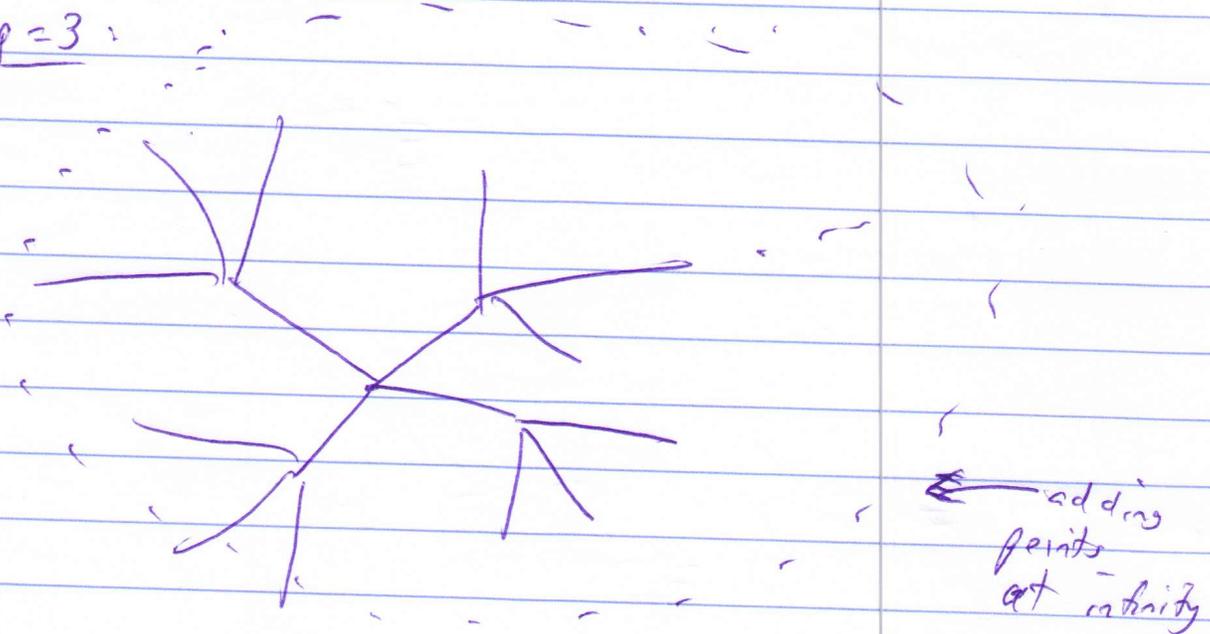
A

Exercise: $U(2, p)$ acts on

BCV

Consider the building of $GL_2(\mathbb{Q}_p)$:

$p=3$:



We can compactify BCV by adding
 a point at the ~~end~~ end of every
 path out to infinity
 i.e. add $\mathbb{P}^1(\mathbb{Q}_p)$

~~add the set of all Borel subgroups~~
~~to BCV~~, which ~~is~~.

Note: $\{ \text{all Borel subgroups} \} = \mathbb{P}^1(\mathbb{Q}_p)$

So compactification of $B(V)$ is

$$B(V) \cup P'(\mathbb{Q}_p).$$

↑ union

This is the analogue of the fact

that the compactification of

$$\mathcal{H} \text{ is } \overline{\mathcal{H}} = \mathcal{H} \cup P'(\mathbb{R})$$

Bruhat - Tits Building and application to representation theory:

Let G/k . Assume G semisimple for

simplicity. Let C be a field.

Define a chain complex from the building:

$$C_* (B(G), C), \text{ which is } \text{acyclic}$$

~~since $B(G)$ is geodesically contractible,~~

where $C_m(B(G), C) := \bigoplus_{F \in \mathcal{B}_m(G)} C(F)$

$= \bigoplus_{F \in \mathcal{B}_m(G)} \text{Ind}_F^G C$ ✓ i.e. trivial rep'n

$\mathcal{B}_m(G)$

G

This gives a projective resolution of the trivial representation. category of reps of G

More generally, if $V \in \text{Mod}_C(G)$, then get a projective resolution of V

from ~~to~~ ~~C~~ $C_*(B(G), V)$ ~~is~~, which is defined as constant coefficient

~~$C_m(B(G), C, V)$~~

$C_m(B(G), V) = \bigoplus_{F \in \mathcal{B}_m(G)} \text{Ind}_F^G V$

$\mathcal{B}_m(G)$

G

There is work on this

by Schneider - Stuhler and

Meyer - Silleveld

10-30-09

Jean Francois Dat - Part 2,

Cohomological methods in the Langlands program

Recall our analogy:

open unit disk

$$\mathbb{U} = \mathcal{H}$$

$SL(2, \mathbb{R})$

$SO(2, \mathbb{R})$



Bruhat-Tits
Tree

Ω



Want to define Define Ω .

We want to consider $\mathcal{H}^{\pm} = \mathbb{P}^1(\mathbb{C}) \setminus \mathbb{P}^1(\mathbb{R})$.

Drinfeld's idea is to do the same for p -adics:

Define $\Omega^{\pm} := \mathbb{P}^1(\overline{\mathbb{Q}_p}) \setminus \mathbb{P}^1(\mathbb{Q}_p)$

(1)

This is not ~~Zariski~~ Zariski open, so need to
talk about p -adic analytic geometry.

Tate "rigid analytic geometry"

Berkovich, ...

Kubert, ...

(Tate, Berkovich, whatever...)

Whatever the language, (1) " \mathbb{Q}_p -analytic spaces" are
glued from "affinoid spaces". These are
locally ringed "topological spaces".

(2) Affinoid spaces form a category which is
anti-equivalent to "affinoid algebras". (like affine schemes)

Defn: An affinoid algebra is a quotient of the

Tate algebra ~~$\mathbb{Q}_p[T_1, \dots, T_n]$~~

$$\mathbb{Q}_p \langle T_1, \dots, T_n \rangle = \left\{ \sum_{v \in \mathbb{N}^n} a_v T^v : |a_v| \rightarrow 0 \text{ as } |v| \rightarrow \infty \right\}$$

= p -adic completion of $\mathbb{Q}_p[T_1, \dots, T_n]$. (2)

Think of $\mathbb{Q}_p \subset \mathbb{T}_1 \dashrightarrow \mathbb{T}_n$ as "globally analytic

functions on $B^n(0,1)$ "

↑ closed unit ball in n -dim'l space.

③ FGA function

{ fin. gen. \mathbb{Q}_p -schemes } \longrightarrow { \mathbb{Q}_p -analytic spaces }

④ If $U \subset X$ is "open" in X and is analytic,
then U is analytic.

⑤ $X(\mathbb{Q}_p)$ is "closed" in X .

So when we do $\Omega^1 = P^1(\overline{\mathbb{Q}_p}) \setminus P^1(\mathbb{Q}_p)$,
we are removing a closed set $P^1(\mathbb{Q}_p)$ from
 $P^1(\overline{\mathbb{Q}_p})$, thus we get an analytic structure on Ω^1 .

③

④

Have map

$$\begin{array}{ccc}
 \mathbb{P}(V)^{\text{an}} & \xrightarrow{\varphi} & \{\text{Seminorms on } V\} \\
 x & \longmapsto & x/V \\
 \cup & & \cup
 \end{array}$$

$$\Omega(V) \xleftrightarrow{\varphi|_{\Omega(V)}} \text{Norms}(V)$$

From $\hat{\Pi}$, get an affinoid covering.

Ex: $V = \mathbb{Q}_p^2, W = \mathbb{Z}_p^2$

$$\Pi^{-1}([w]) = B(0,1) \cup \bigcup_{a \in \mathbb{Z}_p} B(a,1)$$

open unit ball around a

closed unit ball around 0

$$= \text{affinoid space } A_w = \mathbb{Q}_p \langle T, U_0, \dots, U_{p-1} \rangle$$

$$\{T - iU_{p-1}\}$$

$$\Pi^{-1}([w, w^*]) = B(0,1) \cup \bigcup_{b \in \mathbb{Z}_p} B(b, p^{-1})$$

= affinoid space

Now, Cohomology. This is computed by

Schneider-Stuhler :

Fix a basis of V , so $V \cong \mathbb{Q}_p^n$.

So $GL(V) \cong GL(n, \mathbb{Q}_p)$. Let

$I \subset \{1, \dots, n-1\}$. Associate to I ,

$$N_I = \begin{pmatrix} 1 & & & \\ & \ddots & & \\ & & 1 & \\ & & & \ddots \\ & & & & 1 & \\ & & & & & \ddots \\ & & & & & & 1 & \\ & & & & & & & \ddots \\ & & & & & & & & 1 & \\ & & & & & & & & & \ddots \\ & & & & & & & & & & 1 \end{pmatrix}$$

$N_I =$ Jordan form matrix with Jordan blocks of size $1, 2, 3, \dots, n-1$ with ones on the diagonal. i.e.

$$N_I = \begin{pmatrix} \square & & & \\ & \square & & \\ & & \square & \\ & & & \square \end{pmatrix}$$

R coefficient ring.

$$\prod_I (R) = C^\infty(G/P_I, R)$$

$$\sum_{Q \supset P_I} C^\infty(G/Q, R)$$

If $R = \mathbb{Q}_\ell$, it's

known that

$\Gamma \curvearrowright \prod_I (\bar{\mathbb{Q}}_\ell)$ is a bijection

$\{1, \dots, n-1\}$
2

\longleftrightarrow Jordan-Holder factors

in $C^\infty(G/B, \bar{\mathbb{Q}}_\ell)$

Etale ~~sheaf~~ ^{Cohomology}

$H_c^*(\Omega)$ is 0 for $*$ $< n-1$
or $*$ $> 2n-2$



meaning $H_c^*(\Omega \otimes \bar{\mathbb{Q}}_p \otimes \bar{\mathbb{Q}}_\ell)$

$n-1+i$

$$H_c^{n-1+i}(\Omega) \cong \prod_{\{1, \dots, i\}} (\mathbb{Z}_\ell) \otimes \mathbb{Z}_\ell(i)$$

\uparrow Tate twist.

De Rham cohomology: $H_{dR}^* = 0$ if $*$ $> n-1$ and

$$H_{dR}^{n-1+i}(\Omega) \cong \left[\prod_{\{1, \dots, i\}} (\mathbb{Q}_p) \right]^* \leftarrow \text{dual}$$

A priori, this does not seem to be related to Langlands Correspondence:

Well, we have

$$H_c^* \longleftarrow GL_n(\mathbb{Q}_p) \times W_{\mathbb{Q}_p}$$

Weil group
↓

Our dream is that

$$H_c^* = \bigoplus_{\pi \in \{\text{Irrred reps of } G\}} \pi \otimes \sigma(\pi)$$

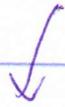
where $\sigma(\pi)$ is the repⁿ of $W_{\mathbb{Q}_p}$ associated to π from LLC.

ie. That H_c^* would be the "graph" of LLC.

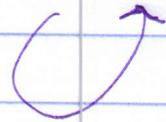
Variants

Consider

$$R\Gamma_c(\mathcal{R} \otimes_{\mathbb{F}_p} \overline{\mathbb{F}_p}, \overline{\mathbb{F}_p}) \in D_{\overline{\mathbb{F}_p}}^b(\mathrm{GL}_n(\mathbb{F}_p))$$



is split



$\mathrm{Gal}(\overline{\mathbb{F}_p}/\mathbb{F}_p)$

Rep'n Theory \Rightarrow can compute everything

Thm: $R^* \mathrm{Hom}(R\Gamma_c, \overline{\mathbb{F}_p}) \cong \mathbb{F}_p^n$ with

action given by Frobenius acts as $\begin{pmatrix} 1 & & \\ & \dots & \\ & & p^{-1} \end{pmatrix}$ and
 inertia acts via $\rho \in I_{\mathbb{F}_p} \mapsto \exp(N_I \rho(i))$

$$\forall I \subset \{1, \dots, n-1\}$$

$$\text{So } \cong \mathcal{L}(\overline{\mathbb{F}_p})$$

~~The pattern of classification of $\mathrm{Irr}(\mathrm{GL}_n)$~~

Drinfeld: He has constructed a tower of

G -equivariant Galois étale coverings

$$\dots \rightarrow M_n \rightarrow M_{n-1} \rightarrow \dots \rightarrow M_0 \rightarrow \Omega$$

$$\text{s.t. } \text{Gal}(M_n/\Omega) \cong \mathcal{O}_D^\times / (1 + \mathfrak{p}^n \mathcal{O}_D)$$

where D is the division algebra

$$\mathcal{O}_{\mathbb{F}_q}[\pi]$$

$$\left\{ \begin{array}{l} \pi^n = \rho \text{ and } \pi x \pi^{-1} = \text{Frob}(x) \\ \text{if } x \in \mathcal{O}_{\mathbb{F}_q} \end{array} \right\}$$

Then would now consider $\varprojlim M_n =: M_\infty$
instead of Ω , as the analogue

Local Approach: ~~Moduli space~~ space of \mathcal{H}

Moduli space for "p-adic groups"
and structure

Global Approach: Uniformization ~~Theory~~ Theory
and global moduli space.

Uniformization theory:

- If C is a projective algebraic curve with genus $g(C) > 1$, then

$$C^{\text{analytic}} = \mathbb{H}^{\pm} / \Gamma \quad \text{with}$$

$$\Gamma \subseteq \text{GL}(2, \mathbb{R}) \quad \text{compact mod center and discrete}$$

- Arithmetic way of producing such Γ .

Start with quaternionic \mathcal{D} over \mathbb{Q}
s.t. $\mathcal{D} \otimes \mathbb{R} \cong M_2(\mathbb{R})$ (i.e.

split at ∞)

Then by strong approximation theory,

$$\text{GL}(2, \mathbb{R}) / \mathcal{O}_{\mathcal{D}}^{\times} \mathbb{R}^{\times} \quad \text{is compact and } \mathcal{O}_{\mathcal{D}}^{\times} \text{ is discrete.}$$

Therefore, writes $S_{\mathcal{D}} := \mathbb{H}^{\pm} / \Gamma_{\mathcal{D}}$ where

$$\Gamma_{\mathcal{D}} = \mathcal{O}_{\mathcal{D}}^{\times}, \quad \text{then}$$

\mathcal{J}_g is algebraizable to some
Complex ~~projective~~ projective curve.

Fact: \mathcal{J}_g has a canonical model over \mathbb{Q} .

More precisely, $\mathcal{J}_g(\mathbb{C})$ parameterizes
pairs (A, i) where A is an abelian
surface and $i: \mathcal{O}_{\mathcal{J}_g} \hookrightarrow \text{End}(A)$

Namely, the parameterization is

$$z \in \mathcal{H} \rightsquigarrow \Gamma_z := \mathcal{O}_{\mathcal{J}_g}^{\bullet}(z),$$

a lattice in \mathbb{C}^2

$\rightsquigarrow \mathbb{C}^2 / \Gamma_z$ together with a natural
~~action of~~ action of $\mathcal{O}_{\mathcal{J}_g}$.

p -adic uniformization

Let C be an algebraic projective curve
over \mathbb{Q}_p .

Question: When does $\exists \Gamma$ s.t.

$$C^{\text{analytic}} \xrightarrow{\sim} \Omega / \Gamma \quad \text{where}$$

$\Gamma \subset GL_2(\mathbb{Q}_p)$ is discrete, compact.

One necessary condition is that C has
a regular integral model \mathcal{C} such that the
special fiber $\mathcal{C} \otimes \mathbb{F}_p$ is a normal

crossing divisor with rational components \mathbb{P}^1 .

Ω has a ~~natural~~ natural formal model $\hat{\Omega}$

such that $\hat{\Omega}_{\mathbb{F}_p}$ is a tree in \mathbb{P}_p .

Thm (Cerednik - Drinfel'd):

Assume $\mathcal{L} \otimes \mathbb{Q}_p$ is a quaternionic algebra.

Then $\int_{\mathcal{L}}$ admits a uniformization by Ω .

More precisely, define another quaternion algebra

\mathcal{L}^* with discriminant $\text{disc}(\mathcal{L}^*)$

satisfying $\text{disc}(\mathcal{L}^*) = \text{disc}(\mathcal{L}) / p$,

$$\mathcal{L}^* \otimes \mathbb{Q}_\ell = \mathcal{L} \otimes \mathbb{Q}_\ell \quad \forall \ell \neq p,$$

$$\mathcal{L}^* \otimes \mathbb{Q}_p = M_2(\mathbb{Q}_p),$$

$$\mathcal{L}^* \otimes \mathbb{R} = \text{quaternionic}.$$

Choose a maximal order $\mathcal{O}_{\mathcal{L}^*}$.

Then $GL_2(\mathbb{Q}_p) / \mathcal{O}_{\mathcal{L}^*}^\times \mathbb{Q}_p^\times$ is compact

and $\Gamma_{\mathcal{L}^*} = \mathcal{O}_{\mathcal{L}^*}^\times$ is discrete
in $GL(2, \mathbb{Q}_p)$,

and $\int_{\mathcal{L}}^{\text{analytic}} \cong \Omega / \Gamma_{\mathcal{L}^*}$.

So the relationship between \mathcal{H} and \mathcal{R} is that they both have uniformization ~~theory~~ theory.

We get a tower of étale groups

$A[p^n]$ over $\mathcal{R}/\Gamma_{\mathcal{D}^*}$, $A[p^n] = \text{kernel}$ ~~of~~ of multiplication by p^n .

whose fibers are $\cong \mathcal{O}_D^* / (1 + p^n \mathcal{O}_D)$
 where $D \cong \mathcal{D} \otimes \mathbb{Q}_p$

So we get a tower of Galois étale coverings with Galois groups

$\mathcal{O}_D^* / (1 + p^n \mathcal{O}_D)$ over $\mathcal{R}/\Gamma_{\mathcal{D}^*}$
 $\uparrow p^n$

Then, pull this tower back to Ω

Can now look at Cohomology complex

$$R\Gamma_c(M_\infty, \overline{\sigma}) \in D_{\sigma}^b(GL_n(\mathbb{Q}_p))$$

$W_{\mathbb{Q}_p} \times D^*$

Thm = ~~...~~ $\forall \pi \in \text{Irr}_{\sigma}(GL_n)$

$$R^* \text{Hom}(R\Gamma_c(M_\infty), \pi) \cong \mathcal{L}(\pi) \otimes \mathcal{L}\mathcal{J}(\pi)$$

Jacquet-Langlands correspondence.

Recall picture

$$\begin{array}{ccc} \mathcal{U} & = & \mathcal{H} \\ \parallel & & \parallel \\ SL(2, \mathbb{R}) / SO(2, \mathbb{R}) & & \end{array}$$

M_∞



Ω



Bracket into pildis

(16)

What is the analogue of \mathcal{U}_p in p -adic?

Well, ~~so~~ literally consider \mathcal{U}_p , the open unit ball around 0, $B(0,1)$ in p -adics.

The \mathcal{U}_p ~~so~~ can be thought of as

the ~~so~~ generic fiber of deformation space of "Lubin-Tate groups of height 2" ~~so~~ ~~on~~ $\overline{\mathbb{F}_p}$

$$\text{End}(X) \cong \mathcal{O}_D$$

$$\mathcal{O}_D^{\times} \hookrightarrow B^{\circ}(0,1) \quad \text{so } \text{looking again}$$

at p^n kernels, get a tower of étale coverings.

$$\dots \rightarrow \tilde{M}_n \rightarrow \tilde{M}_{n-1} \rightarrow \dots \rightarrow \mathcal{U}_p \quad (17)$$

$\underbrace{\hspace{15em}}_{GL_n(\mathbb{Z}_p) / 1 + p^m M_n(\mathbb{Z}_p)}$

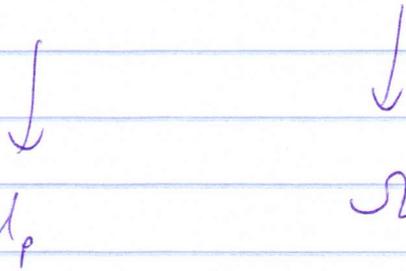
Then define $\tilde{M}_{LT, \infty} := \varprojlim_i \tilde{M}_i$

Then

Faltings-Lagarias $R^* \text{Hom}(R\Gamma_c(\tilde{M}_{LT, \infty}), \pi) \cong L(\pi) \otimes L^J(\pi)$

Picture:

$$\tilde{M}_{LT, \infty} = M_{\infty}$$



$$\mathcal{U} \cong \mathcal{H} \\ \cong \mathcal{U}(2, \mathbb{R}) / \mathcal{S}_0(2, \mathbb{R})$$

Building

He actually denoted \tilde{M}_i by $M_{LT, i}$ and $\tilde{M}_{LT, \infty}$ by $M_{LT, \infty}$, I wrote

the notation incorrectly.